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Interconnect Delay Metrics Using Nakagami Distribution

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ABSTRACT
In deep sub-micrometer regime the interconnect delay dominates over the gate delay. Several attempts have been made to estimate the interconnect delay accurately and efficiently. By interpreting the impulse response of a linear circuit as a Probability Distribution Function (PDF), Elmore first estimated the interconnect delay. Several other approaches like PRIMO, AWE, h-Gamma, WED, D2M etc. have been reported so far, which are shown to be more accurate delay estimation compared to Elmore delay metric. But they suffer from computational complexity when using in the total IC design processes. Our work presents a closed form formula for interconnect delay. Our metrics are derived from matching circuit moments to the Nakagami distribution. The delay metrics can be easily implemented for both step and ramp inputs by using a single look-up table. Experiments validate the effectiveness of the metrics for nets from a real industrial design.

Categories and Subject Descriptors
B.7.2 [**Integrated Circuits**]: Design Aids – Simulation;

General Terms
Algorithms, Design, Theory

Keywords
Delay calculation, Distribution function, Interconnect, Moment matching

1 INTRODUCTION
As the process technology shrinks into nanometer regime, interconnect delay dominates over the gate delay; hence interconnect delay computation is becoming the crucial bottleneck for both performance and physical design optimization for high speed integrated circuits. The Elmore Delay [1] which is the first moment of the impulse response provides standard delay estimation for performance driven design applications. Elmore approximated the median of the impulse response (50% delay of the step response) by the mean of the impulse response by noting the similarity between non-negative impulse response and probability density functions. The Elmore delay metric has been incredibly popular because of it's simplicity, closed-form and easy to estimate. The unacceptable loophole of the Elmore metric is that it is highly inaccurate as it doesn't consider the resistive shielding effect of the interconnect.

In order to estimate the delay accurately and efficiently several works have been reported so far. Rubenstein et al. [2] proposed a simple closed-form formula for computing the mean of the impulse response of RC interconnect trees. Alpert et al. [3] proposed the D2M metric which is a simple function of the first two circuit moments. The PRIMO [4], h-Gamma [5], and WED [6] metrics are based on matching the moments of the impulse response to a particular continuous probability distribution function (PDF). PRIMO and h- Gamma match moments of the impulse response to the Gamma distribution, while WED matches to the Weibull distribution. The approaches proposed in [4-6] require some type of table lookup operation. In order to improve the accuracy of the Elmore delay metric Asymptotic Waveform Evaluation (AWE) is being proposed [9] by matching the higher order moments of the impulse response. As the technology is shrinking towards the ultra deep sub micrometer (DSM) regime and transistor density in the chip is increasing, the length of the interconnect is getting longer. So efficient and accurate computation of the interconnect delay has become increasingly critical.

We present a closed form delay metrics based on the Nakagami distribution. Unlike [4] [5] [6], matching to the Nakagami distribution produces closed form formulae and no look-up table is required to compute the delay.

We make the following contributions:

A simple delay metric NkD (Nakagami delay) is derived using the first two moments of the impulse response. The NkD metrics can be extended to ramp inputs using the PERI method [7]. The effectiveness of the Nakagami metrics is confirmed on nets from an industrial design.

2 THEORY
Assume that *h(t)* is the impulse response of a node voltage in an RC circuit. The circuit moments of the impulse response are:

$$m_k = \frac{(-1)^k}{k!}\int_0^\infty t^k h(t)\,dt \qquad (1)$$

Where k=1, 2, 3...

and m_k is the k^{th} circuit moment of the impulse response.

The circuit moments can be computed directly as functions of the RC's in time linear in the size of the circuit, e.g., via path tracing algorithm. From [2] the impulse response *h(t)* satisfies the following conditions:

$$h(t) \geq 0 \text{ and } \int_0^\infty h(t)dt = 1 \quad (2)$$

Consequently, the impulse response is a probability distribution function (PDF), though there is no known underlying statistical distribution describing it.

The mean of the impulse response is:

$$\mu = \int_0^\infty t \cdot h(t)dt \quad (3)$$

Elmore [4] showed that $\mu = -m_1$ and therefore approximated the median (the desired delay) by the mean of the impulse response. We let ED = $\mu = -m_1$, denote the Elmore delay. The k^{th} central moment is given by:

$$\mu_k = \int_0^\infty (t-\mu)^k h(t)dt \quad (4)$$

The variance (σ^2) of the impulse response can be expressed in terms of the central moments and also the circuit moments [10]:

$$\sigma^2 = \mu_2 = 2m_2 - m_1^2 \quad (5)$$

The key idea behind our delay metrics is to match the mean and variance of the impulse response to those of the Nakagami distribution.

3 PROPOSED DELAY METRIC

The Nakagami distribution is a two-parameter continuous distribution. The Nakagami distribution is well suited to match the impulse response since both are unimodal and have nonnegative skewness. The Nakagami PDF is given by:

$$P(x,\mu,w) = \frac{2\mu^\mu}{\Gamma(\mu)\, w^\mu} x^{2\mu-1} \exp(-\frac{\mu}{w}x^2) \quad (6)$$

Where w>0 and μ > 0 are the scale and shape parameters, respectively. Its cumulative density function (CDF) is given by:

$$D(x,\mu,w) = \frac{\gamma(\mu, \frac{\mu}{w}x^2)}{\Gamma(\mu)} \quad (7)$$

The expected value (or mean) and the variances are, respectively, given by:

$$E(x) = \frac{\Gamma(\mu+\frac{1}{2})}{\Gamma(\mu)}(\frac{w}{\mu})^{1/2} \quad (8)$$

and $$V(x) = w\left(1 - \frac{1}{\mu}\left(\frac{\Gamma(\mu+\frac{1}{2})}{\Gamma(\mu)}\right)^2\right) \quad (9)$$

One can match two common properties of the Nakagami distribution and the circuit's impulse response. Recall that the mean and variance of the impulse response are $\mu = -m_1$ and $\sigma^2 = 2m_2 - m_1^2$, respectively. Using Equation (8) and (9) to match the mean and variance yields,

$$\frac{\Gamma(\mu+\frac{1}{2})}{\Gamma(\mu)}(\frac{w}{\mu})^{1/2} = -m_1 \quad (10)$$

and $$w\left(1 - \frac{1}{\mu}\left(\frac{\Gamma(\mu+\frac{1}{2})}{\Gamma(\mu)}\right)^2\right) = 2m_2 - m_1^2 \quad (11)$$

Solving (10) and (11) for '*w*' yields,

$$w = \left[\frac{(2m_2 - m_1^2) + \sqrt{\left(2m_2 - m_1^2\right)^2 + 4m_1^2}}{2}\right] \quad (12)$$

Note that the median of the Nakagami Distribution is given by $\sqrt{w}$. One can verify this by setting $D(x,\mu,w)$ = 0.5 in Equation (7) and solving for *x*. Thus, when matching the impulse response the median becomes our 50% delay metric:

$$NkD = \sqrt{w} = \sqrt{\left(\frac{(2m_2 - m_1^2) + \sqrt{\left(2m_2 - m_1^2\right)^2 + 4m_1^2}}{2}\right)} \quad (13)$$

Thus, the delay function is a simple function of the first two circuit moments. This is our proposed closed form delay model.

4 EXPERIMENTS RESULTS

In order to verify of the effectiveness of our model we have extracted 432 routed nets containing 2244 sinks from an industrial ASIC part in 0.18 micrometer technology. The nets were chosen by filtering process that required the maximum sink delay to be at least 10ps and the ratio of the closest sink to the furthest sink in the net to be less than 0.25. So, each net has at least one near-end sink. Now for each sink we compute delay using SPICE and measure the relative error of the appropriate metric to the SPICE result. For each RC network source we put a driver, where the driver voltage is a voltage source followed by a resister. Here we have compared Nakagami Distribution (NkD) with Kahng-Muddu Model (KM) [8] and Elmore Delay (ED) [1].

Table 1. Delay comparison for Nakagami delay metric

Driver Resistance=0Ω						
	Average % Relative error			% Standard Deviation		
Sinks	NkD	ED	KM	NkD	ED	KM
Near	56.7	335.9	141.8	42.34	207.98	103.63
Mid	18.4	87.33	19.98	12.98	34.09	15.37

Far	1.29	33.64	1.31	1.19	3.58	1.65
Total	18.13	113.7	37.18	31.13	148.6	74.76
Driver Resistance = 100Ω						
Sinks	NkD	ED	KM	NkD	ED	KM
Near	109.5	521.1	246.7	67.89	307.87	164.76
Mid	18.2	81.23	18.89	11.34	28.03	14.49
Far	1.49	36.56	1.65	1.13	3.79	1.29
Total	26.76	143.2	52.6	49.67	229.76	117.87
Driver Resistance = 200Ω						
Sinks	NkD	ED	KM	NkD	ED	KM
Near	123.76	457.9	228.4	62.98	228.48	222.12
Mid	14.89	74.53	15.23	8.47	23.12	14.11
Far	1.65	34.67	1.69	0.87	3.67	1.47
Total	27.67	126.6	45.43	52.34	189.1	45.19

5 CONCLUSION

We have proposed NkD, a closed form delay metric for RC trees that is a simple function of two moments of the impulse response, for performance optimization. Our metric has the Elmore delay as a theoretical upper bound, but with significantly less error. NkD is more accurate than KM and is indeed remarkably accurate at the near end. NkD has the advantage that its Elmore-like formula may make it more amenable to optimization.

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