Structural Modeling and Analysis of Computer Aided Diagnosis (CAD) system: A Graph Theoretic Approach

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ABSTRACT

The main purpose of this paper is to document the research on development of a conceptual framework for Computer Aided Diagnosis (CAD) system. An attempt is made to develop an integrated system model for the Structural modeling and analysis of CAD system in terms of its constituents and interpretation of medical images using graph theory and matrix algebra. The CAD system is first modeled with the help of a graph theory, then by a variable adjacency matrix and then by a multinomial known as a permanent function. The permanent function provides an opportunity to carry out structural analysis of the CAD system in terms of strength, weakness, improvement, and optimization in detection and precise characterizations of potential abnormalities which in turn enables clinical step towards effective treatment. A physical meaning has been associated with each term of the permanent function. Different structural attributes of the CAD subsystems are identified to develop a graph theoretic model, a matrix model, and a multinomial permanent model of the CAD. A top-down approach for complete analysis of any CAD product system is also given. A general methodology is also presented for characterization and comparison of CAD systems. Usefulness of the present methodology is also illustrated.

Keywords

CAD, digraph approach, structural modeling, permanent function, CAD system.

1 INTRODUCTION

The accurate quantification of disease patterns in medical images allows radiologists to track the progress of the disease. Image analysis is still performed manually, which is often a difficult and time-consuming task. Consequently, there is an increasing need for computerized image analysis to facilitate image based diagnosis. So, many investigators have carried out basic studies and clinical applications toward the development of modern computerized schemes called Computer Aided Diagnosis system for detection and characterization of lesions in images such as brain, chest, colon, breast, liver, kidney and the vascular and skeletal systems, based on computer vision and artificial intelligence systems Early detection is the most effective way to reduce mortality. This leads a crucial step towards the institution of effective treatment [5,8]. Most of the radiologist achieve this goal by the process of image perception to recognize unique image pattern and then to identify relationships between perceived patterns and possible diagnosis. But both detection and characterization processes depend heavily on the radiologists' empirical knowledge, memory, intuition and diligence. Hence they approach the diagnostic task with a level of intelligence, flexibility and common sense. So there are chances for well documented errors and variations in the human interpretation of clinical images .Indeed, estimates indicate that between 10 and 30% of breast cancers are missed by radiologists during routine screening [1,2]. The goal of the intelligent system is to provide the radiologists with a second opinion on a lesion diagnosis to achieve high accuracy and save the human lives.

From the literature review, no one has considered the structural constituents for analyzing a medical diagnosis system along with their interactions at the conceptual and design stages, and also there is no methodology proposed for an integrated system approach for analyzing the CAD by considering the subsystems as its constituents, and their interactions, interdependence, and connectivity. There is a need to develop a mathematical model that can help in analysis and synthesis of a CAD system at the conceptual stage and also in developing a virtual model. In the present scenario, the virtual fabrication adds several advantages to medical imaging, automobile and aircraft industry for improving its product design like faster production rate, achieving less cost, getting good theoretical results without experimentation for better product quality, etc. We propose a new integrated systems approach that can help in getting good compelling benefits, high overall performance, and good quality of accuracy in a shorter time. Therefore, it is expected that CAD schemes will be assembled in the future as a package for detection of lesions and also for differential diagnosis which could be employed as a useful tool for diagnostic examinations in daily clinical work.

2 IDENTIFICATION OF SUB SYSTEMS OF CAD SYSTEM

To develop the system mathematical model of the structure of CAD system, it is necessary to understand not only the structural constituents but also their pre and post processing operations, which ultimately produce useful CAD products with a wide variety of medical applications. The texture features play an essential role in identifying the lesions in the medical image. Image texture, defined as a function of the spatial variation in pixel intensities (gray values), which is useful in a variety of applications and has been a subject of intense study by many researchers. Texture is a rich source of visual information and is a key component in image analysis and understanding in humans.

Texture is known to provide cues about scenes depth and surface orientation. It describes the content of both natural artificial images with reality. There is evidence of perceptual learning in texture coding mechanisms [3] and in textural discrimination.

Different systems of CAD are classified in five groups as:S_1 = Preprocessing and Enhancement subsystem [PES] :S_2 = Segmentation Subsystem [SS]:S_3 = Feature extraction Subsystem [FS]:S_4 = Classification Subsystem [CS]:S_5 = Performance Analysis & Product Design Subsystem[PAPS]

The Sub subsystems are, given as follows:

PES=Image Acquisition, Artifacts Removal, Filtering, Normalization, Pectoral Removal, Enhancing the Image.

SS= Single Image segmentation, Labeling, Thresholding.

FS=Co occurrence matrix, Gray level calculation, Heralick Feature, Feature Selection.

CS=Samples, Learning Topology, Training, Algorithm Selection, Sensitivity, Specificity

PAPS=Shape and Size ,Complexity, Technical Support, Cost Effective, In-depth clinical Application Knowledge, Continuous innovation.

Each of these five subsystems S_i , i = 1,.., 5 is identified by a hierarchical tree to indicate its contribution and scope through its sub-sub systems in total CAD System. Each of these hierarchical trees need to be replaced by corresponding block diagram showing interaction between its systems, as each subsystem is a system in itself.

3 BLOCK DIAGRAM REPRESENTATION

The CAD System is considered to be a collection of a number of basic structural constituents, S_i , i = 1,2,3,4,5. These constituents are connected with each other through different forms of bonding and interactions. The constituents and interactions forming a CAD System are shown in Figure 1 with the help of a schematic diagram. Blocks show constituents, lines show connectivity/interaction, and arrows show directional bonding/interaction. However, schematic diagram is a good representation of CAD System for a better understanding of its structure but it is not a mathematical entity [4].

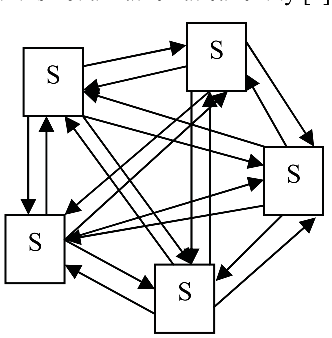

Figure 1 Block diagram of CAD System showing interactions

3.1 GRAPH THEORETIC REPRESENTATION

A CAD system may be considered to be a system [S,I] of its constituent set {S} = {S_1, S_2, . . . , S_n} and interaction/connectivity set {I} = {I_1, I_2, . . . , I_n}, where S_i represents i^{th} constituent (and structure associated with it) while I_j represents j^{th} interaction/connectivity between two corresponding constituents of the structure. For the systems representation of the structure, it is quite logical to select graph theory [4] .To develop an algorithm the constituent S_i is represented by the vertex Si. Let any connectivity/interaction existing between the constituents Si and Sj, i.e., between vertices Si and S_j be represented by the edge S_{ij} (an edge existing between vertices Si and S_j). If we assume that all the five constituents are interacting with each other and have general directional characteristics, the CAD has a graph theoretic representation with $S_{ij} \neq S_{ji}$. The $S_{ij} \neq S_{ji}$ means that the influence of ith vertex on jth vertex is not equal to the influence of j^{th} vertex on ith vertex. If the directional property is not significant, the CAD System is represented by an undirected graph, in this case $S_{ij} = S_{ji}$.

3.2 MATRIX Representation

Let us represent a digraph of 'n' subsystems nth-order symmetric (0, 1) adjacency matrix A = [S_{ij}]. The rows and columns in the matrix represent the vertices/subsystems, i.e., S_{ij} represents the interaction/connectivity of the ith subsystem with the jth subsystem:

$$S_{ij} = \begin{cases} 1, \text{ if subsystem i is connected to subsystem j} \\ 0, \text{ otherwise} \end{cases}$$

The CAD System matrix is square and symmetric and is represented s follows.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Equation (1) stores connectivity information of the system only. To characterize the CAD system, a characteristic matrix is defined.

4 CAD- CHARACTERISTIC MATRIX (CAD-CM)

Let us consider an identity matrix I, and S as the variable representing subsystems of the CAD System. The characteristic matrix is used to characterize the CAD System.

$$B = \begin{bmatrix} S & -1 & -1 & -1 & -1 \\ -1 & S & -1 & -1 & -1 \\ -1 & -1 & S & -1 & -1 \\ -1 & -1 & -1 & S & -1 \\ -1 & -1 & -1 & -1 & S \end{bmatrix} \tag{2}$$

Determinant of matrix B gives

Det (B) = S^5-10xS^3-20xS^2-15xS-4
(2a)

Determinant of characteristic matrix B equation e.g. (2a) is called characteristic polynomial and characteristic of the CAD System. It is powerful characterizing invariant of the system. At this stage the problem is now considered from combinational / discrete mathematics point of view. In the above matrix B, the value of all diagonal systems is the same Interdependencies between the subsystems have been assigned values of 0 and 1 depending on whether it is there or not. This does not represent varying degree of influence of one subsystem over the other subsystems and also distinct function of these five subsystems. To consider this, another matrix called the variable characteristic matrix is proposed.

4.1 CAD -Variable Characteristic Matrix (CAD -VCM)

Let S_is and S_{ij}s represent nodes and edges, respectively, in the digraph. Consider a square matrix C with off-diagonal systems S_{ij} representing varying interactions between the CAD subsystems, i.e., instead of 1 (as in the matrix of Equation (1)). Another matrix D is taken with diagonal systems S_i, i=1, 2, 3, 4, 5 where the S_i represents five different subsystems. As these are distinct systems, S_i will represent varying inheritance of structural attributes in these subsystems. Considering matrices C and D,

CAD VCM is expressed as H = [D-C]

$$H=\begin{bmatrix} S_1 & -S_{12} & -S_{13} & -S_{14} & -S_{15} \\ -S_{21} & S_2 & -S_{23} & -S_{24} & -S_{25} \\ -S_{31} & -S_{32} & S_3 & -S_{34} & -S_{35} \\ -S_{41} & -S_{42} & -S_{43} & S_4 & -S_{45} \\ -S_{51} & -S_{52} & -S_{53} & -S_{54} & S_5 \end{bmatrix} \tag{3}$$

The above matrix, H, permits to represent complete information about all the five subsystems and interactions amongst. The matrix provides a powerful tool through its determinant, called the variable characteristic multinomial this is a characteristic of the system and represents the complete CAD, considering the effects its subsystems and their interactions. But the Equation (3), does not retain complete information concerning the CAD System under certain conditions but in symbolic notations this multinomial is definitely unique characteristics equation. To avoid the loss of structural information during mathematical processing, another term Variable Permanent Matrix (CAD-VPM) is introduced.

4.2 CAD -Variable Permanent Matrix (CAD -VPM)

To develop a unique and comprehensive model of CAD system, another entity permanent matrix, frequently used in combinatorial mathematics is proposed [7] . Let the permanent matrix of five-subsystem CAD be defined as,

$$\begin{aligned}
per(H) = {} & S_1S_2S_3S_4S_5 + (S_1S_2S_3S_{45}^2 + S_1S_2S_4S_{35}^2 + S_1S_2S_5S_{34}^2 + S_1S_3S_4S_{25}^2 \\
& + S_1S_3S_5S_{24}^2 + S_1S_4S_5S_{23}^2 + S_2S_3S_4S_{15}^2 + S_2S_3S_5S_{14}^2 + S_2S_4S_5S_{13}^2 + S_3S_4S_5S_{12}^2) \\
& + (2S_1S_2S_{34}S_{45}S_{53} + 2S_1S_3S_{24}S_{45}S_{52} + 2S_1S_4S_{23}S_{35}S_{52} + 2S_1S_5S_{23}S_{34}S_{42} + 2S_2S_3S_{14}S_{45}S_{51} \\
& + 2S_2S_4S_{13}S_{35}S_{51} + 2S_2S_5S_{13}S_{34}S_{41} + 2S_3S_4S_{12}S_{25}S_{51} + 2S_3S_5S_{12}S_{24}S_{41} + 2S_4S_5S_{12}S_{23}S_{31}) \\
& + [(S_1S_{23}^2S_{45}^2 + S_1S_{24}^2S_{35}^2 + S_1S_{25}^2S_{34}^2 + S_2S_{13}^2S_{45}^2 + S_2S_{14}^2S_{35}^2 + S_2S_{15}^2S_{34}^2 + S_3S_{12}^2S_{45}^2 + S_3S_{14}^2S_{25}^2 + S_3S_{15}^2S_{24}^2 \\
& + S_4S_{12}^2S_{35}^2 + S_4S_{13}^2S_{25}^2 + S_4S_{15}^2S_{23}^2 + S_5S_{12}^2S_{34}^2 + S_5S_{13}^2S_{24}^2 + S_5S_{14}^2S_{23}^2) + (2S_1S_{23}S_{34}S_{45}S_{52} \\
& + 2S_1S_{23}S_{35}S_{54}S_{42} + 2S_1S_{24}S_{43}S_{35}S_{52} + 2S_2S_{13}S_{34}S_{45}S_{51} + 2S_2S_{14}S_{43}S_{35}S_{51} + 2S_2S_{15}S_{53}S_{34}S_{41} \\
& + 2S_3S_{12}S_{24}S_{45}S_{51} + 2S_3S_{14}S_{42}S_{25}S_{51} + 2S_3S_{12}S_{25}S_{54}S_{41} + 2S_4S_{12}S_{23}S_{35}S_{51} + 2S_4S_{13}S_{32}S_{25}S_{51} \\
& + 2S_4S_{12}S_{25}S_{53}S_{31} + 2S_5S_{13}S_{32}S_{24}S_{41} + 2S_5S_{12}S_{23}S_{34}S_{41} + 2S_5S_{13}S_{34}S_{42}S_{21})] + [(2S_{12}^2S_{34}S_{45}S_{53} \\
& + 2S_{13}^2S_{24}S_{45}S_{52} + 2S_{14}^2S_{23}S_{35}S_{52} + 2S_{15}^2S_{23}S_{34}S_{42} + 2S_{23}^2S_{14}S_{45}S_{51} + 2S_{24}^2S_{13}S_{35}S_{51} + 2S_{25}^2S_{13}S_{34}S_{41} \\
& + 2S_{34}^2S_{12}S_{25}S_{51} + 2S_{35}^2S_{12}S_{24}S_{41} + 2S_{45}^2S_{12}S_{23}S_{31}] + [2S_{12}S_{23}S_{35}S_{54}S_{41}) + (2S_{12}S_{23}S_{34}S_{45}S_{51} + 2S_{12}S_{24}S_{45}S_{53}S_{31} \\
& + 2S_{12}S_{24}S_{43}S_{35}S_{51} + 2S_{12}S_{25}S_{53}S_{34}S_{41} + 2S_{12}S_{25}S_{54}S_{43}S_{31} + 2S_{13}S_{32}S_{24}S_{45}S_{51} + 2S_{13}S_{34}S_{42}S_{25}S_{51} \\
& + 2S_{13}S_{35}S_{52}S_{24}S_{41} + 2S_{13}S_{32}S_{25}S_{54}S_{41} + 2S_{14}S_{42}S_{23}S_{35}S_{51} + 2S_{15}S_{52}S_{23}S_{34}S_{41})]
\end{aligned} \tag{5}$$

It is possible to write these equations simply by visual inspection of the system of Fig 2 by identification of these subsets. To achieve this objective, the permanent function of Equation (5) is written in a standard form in (N+1) groups.

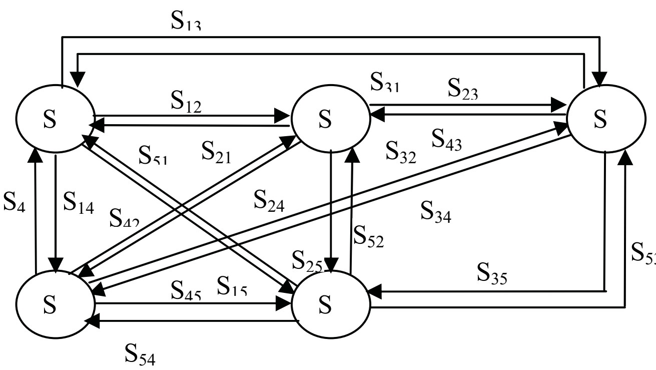

Fig.2 CAD system Digraph

The multinomial, i.e., the permanent function when written in (N+1) groups, presents an exhaustive way of analysis of a System at different levels [6]. It helps in identifying different constituents, process parameters, implementation attributes, and the interactions among various subsystems of CAD. The terms of equation 5 is arranged in (N+1) graph as

1. The first group contains only one term, which is a set of five unconnected vertices S_is.

2. The second group does not have any term as a particular subsystem S_i cannot connect or influence itself.

3. The third group has terms consisting of dyads S_{ij} S_{ji} or S^2_{ij} and the remaining three unconnected vertices S_is (for example $S^2_{12}S_3S_4S_5$).

4. The fourth group consists of a three-vertex loop $S_{ij}S_{jk}S_{ki}$ and the remaining two unconnected vertices S_is (for example S_{12} S_{23} $S_{31}S_4S_5$).

5. The fifth group consists of two subgroups. The first subgroup consists of four-vertex loop S_{ij} S_{jk} S_{kl} S_{li} and one unconnected vertex, while the second subgroup consists of a set of two dyads

S_{ij} S_{ji} and S_{kl} S_{lk} and the remaining one unconnected vertex (for example (S_{12} S_{21}) (S_{34} S_{43}) S_5 and (S_{12} S_{23} S_{34} S_{41})(S_5).

6. The sixth group consists of two subgroups. The first subgroup consists of a five-vertex loop S_{ij} S_{jk} S_{kl} S_{lm} S_{mi} and S_{im} S_{ml} S_{lk} S_{kj} S_{ji} while the second subgroup consists of one dyad $S_{ij}S_{ji}$ or $S^2{}_{ij}$ one 3-vertex loop S_{ij} S_{jk} S_{ki}, and an S_{ik} S_{kj} S_{ji}.

In all, a general 5-subsystem permanent function will have 5!, i.e., 120 terms arranged in (N+1) groups. Every term is a distinct set of structural models (S_i, $S^2{}_{ij}$ and S_{ij} $S_{jk}\ldots S_{mi}$) of Information System. Every term is a subset/subsystem of the total system. Permenant function contains all possible subsystems of CAD System. It is therefore possible for the designer as well as the maintenance expert to carry out SWOT (strength-weakness-opportunities-threats) analysis of their complete CAD system and take strategic decisions to their advantage as per policy.

5 USEFULNESS OF THE PROPOSED METHOD

It helps to develop a variety of CAD system approaches providing optimum performance characteristics under different applications.To assist the development of CAD System the following step-by-step procedure is proposed

Step 1: Consider the desired CAD system. Study the complete system and its subsystems and also their interactions.

Step 2: Develop a block diagram of the System, considering its subsystems and interactions along with assumptions, if any.

Step 3: Develop a systems graph of the System with subsystems as nodes and edges for interconnection between the nodes.

Step 4: Develop the matrix and multinomial representations.

Step 5: Evaluate functions/values of diagonal systems from the permanent functions of distinct subsystems and repeat Steps 2–4 for each subsystem.

Step 6: Identify the functions/values of off-diagonal systems/interconnections at different levels of hierarchy of the information System, which are systems, subsystems, sub subsystems, etc.

Step 7: carry out critical analysis of the system from structural, functional and performance point of view considering one term at a time.

6 CONCLUSION

The proposed systems methodology for developing a Computer Aided Diagnosis System by considering all attributes responsible for development, and implementation, along with interactions between the constituents and using a digraph and matrix approach is a powerful tool for structural modeling CAD system constituents and their interaction with each other. The present work identifies five characteristics/subsystems, which parameterize the CAD system. The systems methodology consists of the CAD System digraph, the CAD system matrix, and the CAD system permanent function. The CAD System digraph is the mathematical representation of the structural characteristics and their interdependence, useful for visual modeling and analysis. The CAD system matrix converts digraph into another mathematical form. This matrix representation is a powerful tool for storage and retrieval of CAD system. The permanent function is a unique mathematical model characterizing the structure of the CAD system irrespective of labeling of subsystems and also helps one to determine the CAD index. The approach integrates all possible structural, functional and performance parameters in a mathematical model for analysis and optimization. An attempt will be made in future publications to correlate CAD systems subsystems' performance characteristics with the CAD structural model reported in this article.

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