N-Dimensional FRW Dust Filled Universe with Time Dependent $\Lambda(t)$ in Creation Field Theory of Gravitation

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Abstract— N-dimensional FRW cosmological model in Hoyle-Narlikar’s creation field theory of gravitation has been studied, when the universe is filled with dust distribution. To get the deterministic solution, the cosmological constant $\Lambda(t)$ is considered as a time dependent function of cosmic time $t$, i.e., $\Lambda = \frac{1}{R^2}$, where $R$ is a scale factor. The physical aspects of the model are also studied.

Keywords—Creation field theory, N-dimensional FRW space time, Varying cosmological constant $\Lambda(t)$.

I. INTRODUCTION

The big bang cosmological model based on Einstein’s field equations is the only model which explains the three important observations in astronomy viz: (i) the phenomenon of expanding universe (ii) primordial nucleo-synthesis (iii) the observed isotropy of the Cosmic Microwave Background Radiations. However Smoot et al. [1] revealed that the earlier predictions of FRW type of models do not always exactly meet our expectations. The theoretical explanations given from big-bang type model were contradicted by some puzzling results regarding the red-shifts from extra-galactic objects. Also CMBR discovery did not prove it to be an outcome of big-bang theory. The possibilities of non-relic interpretation of CMBR have been proved by Narlikaret al. [2]. Alternative theories of gravitation have been proposed by cosmologists to explain such phenomenon. Bondi and Gold [3] proposed the steady state theory in which the universe does not have singular beginning or an end on the cosmic time scale. Moreover they state that the statistical properties of the large scale features of the universe do not change. Further the constancy of matter density has been accounted by continuous creation of matter going on in contrast to one time infinite and explosive creation of matter at $t = 0$ as in earlier standard model. But the principle of conservation of matter was violated in this formalism. This difficulty was overcome by Hoyle and Narlikar [4-6] by adopting a field theoretical approach and introducing a massless & chargeless scalar field $C$ in the Einstein-Hilbert action to explain creation of matter. In Hoyle-Narlikar C-field theory there is no big-bang type singularity as in steady state theory by Bondi and Gold. Narlikar [7] has shown that matter creation is accomplished at the expense of negative energy $C$-field in which he solves horizon and flatness problem faced by big-bang model. Narlikar and Padmanabhan [8] have obtained a solution of Einstein field equations admitting radiation with a negative energy massless scalar field $C$. Chatterjee and Banerjee [9] have investigated higher dimensional cosmology in C-field theory of gravitation. Singh and Chauhey [10] have studied Bianchi type I, III, V, VIo and Kantowski-Sachs universes in creation field cosmology. Adhaver et al. [11] have investigated N-dimensional Bianchi type-V universe in Creation-field Cosmology. Katore [12] has studied plane symmetric universe in C-field theory of gravitation. Recently, Panigrahiet al. [13] investigated five dimensional spherical symmetric universe in Creation field cosmology. Cosmological constant problem is one of the outstanding problems in cosmology [14]. A wide range of observations suggest that universe possesses a non-zero cosmological constant. During an early exponential phase, the vacuum energy is treated as large cosmological constant, which is expected by Glashow-Salam-Weinberg and Grand Unified Theory as mentioned by Langacker [15]. Therefore, the present day observations of smallness of cosmological constant ($\Lambda \leq 10^{-50} \text{cm}^{-2}$) support to assume that the cosmological constant($\Lambda$) is time dependent. Gibbon and Hawking [16] have studied the cosmological model with positive cosmological constant which leads to de-Sitter space time asymptotically. Therefore, the cosmological models linking the variations of cosmological constant are having the form of Einstein’s field equations unchanged and preserving the energy-momentum tensor of matter content. Bertolami [17] was the first who consider cosmological models with a variable cosmological constant of the form $\Lambda \sim r^{-2}$. Chen and Wu [18] have also solved the problem by considering $\Lambda \sim r^{-2}$, where $r$ is the scale factor in the Robertson-Walker space time. Overduin [19] considered the assumptions $\Lambda \sim r^{-2}$ and $\Lambda \sim H^{-2}$ in FRW models and shows there compatibility with various observations. Several attempts have made by many researchers viz. Lui& Wesson [20], Carneiroet al. [21], Ram &Verma [22] and Amirhashchi&Mohamadian [23] in favor of time dependent $\Lambda \sim r^{-2}$ in different contexts. Bali et al. [24-25] have investigated Bianchi type III & FRW cosmological models with varying $\Lambda$ in Creation field cosmology. Chaubeyet al. [26] have obtained general class of Bianchi cosmological models with $\Lambda$
in creation field cosmology. Recently, Ghate and Salve [27-31] have studied cosmological models with varying }\Lambda(t)\text{ in creation field theory of gravitation.}

In this paper, we have investigated N-Dimensional FRW space-time, when the universe is filled with dust distribution. To get the deterministic solution, the cosmological constant }\Lambda(t)\text{ is considered as a time dependent function of cosmic time } t \text{ i.e. } \Lambda = \frac{1}{R^2},

where } R \text{ is a scale factor. This work is organized as follows. In Section 2, the model and field equations have been presented. The solution of field equations with special cases viz. Flat model } (k = 0) \text{, Closed model } (k = 1) \text{ and Open model } (k = -1) \text{ has been discussed in Section 3. In the last Section 4 concluding remarks have been expressed.}

II. METRIC AND FIELD EQUATIONS:

We consider N-Dimensional FRW metric considered in the form

\[ ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 (d\phi_{n-2})^2 \right], \]

where

\[ k = 0, \pm 1 \quad (1) \]

and

\[ (d\phi_{n-2})^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_2 \sin^2 \theta_3 \theta_4 d\theta_3^2 + \cdots + \sin^2 \theta_{n-3} \sin^2 \theta_{n-2} d\theta_{n-2}^2. \]

Here } R(t) \text{ represents the scale factor.}

Einstein field equations in Creation-field theory of gravitation [4-6] with varying } \Lambda(t)\text{ are

\[ R^\prime_{(n)} - \frac{1}{2} Rg^\prime_{(n)} = -8\pi G \left( T^\prime_{(n)} + T^\prime_{(c)} \right) - \Lambda g^\prime_{(n)} \quad (2) \]

The energy momentum tensor } T^\prime_{(n)} \text{ for perfect fluid and } T^\prime_{(c)} \text{ for creation field are given by

\[ T^\prime_{(n)} = (\rho + p)v_i v^i - pg^\prime_{(n)}, \quad (3) \]

and

\[ T^\prime_{(c)} = -f(C_e c^i - \frac{1}{2} R^\prime_{(c)} C^{e a} C_{a i}). \quad (4) \]

Here } \rho \text{ is the energy density of massive particle and } p \text{ is the pressure. } v_i \text{ are co-moving four velocities which obeys the relation } v_i v^i = 1, \quad v_0 = 0, \quad \alpha = 1, 2, 3. \text{ The coupling constant between matter and creation field is greater than zero. It is assumed that creation field } C \text{ is a function of time only i.e. } C(x, t) = C(t). \text{ With the help of equations (3) and (4), the field equations (2) for the metric (1) are

\[ \frac{(n-1)(n-2)}{2} \frac{\ddot{R}}{R^2} + \frac{(n-1)(n-2)}{2} \frac{k^2}{R^2} = 8\pi G \left( \rho - \frac{1}{2} f \dot{C}^2 \right) + \Lambda, \quad (5) \]

\[ 8\pi G \left( -p + \frac{1}{2} f \dot{C}^2 \right) + \Lambda \quad (6) \]

where overhead dot ( ) denotes differentiation with respect to cosmic time } t \text{ .}

The conservation equation

\[ \left( 8\pi G \sqrt{\rho + \Lambda} \right)_i = 0 \quad (7) \]

leads to

\[ 8\pi G \left( \rho - \frac{1}{2} f \dot{C}^2 \right) + 8\pi G \left( \rho - \frac{1}{2} f \dot{C}^2 \right) + \Lambda = 0 \quad (8) \]

Using } G = \text{ constant, equation (8) leads to

\[ 8\pi G \left( \rho - \frac{1}{2} f \dot{C}^2 \right) + 8\pi G \left( \rho - \frac{1}{2} f \dot{C}^2 \right) + \Lambda = 0 \quad (9) \]

The universe is filled with dust distribution, hence using } p = 0 \text{ , equations (6) & (9) reduces to

\[ \frac{(n-1)(n-2)}{2} \frac{\ddot{R}}{R^2} + \frac{(n-1)(n-2)}{2} \frac{k^2}{R^2} = 4\pi G f \dot{C}^2 + \Lambda, \quad (10) \]

\[ 8\pi G \rho - 8\pi G f \dot{C}^2 + 8\pi G \rho \frac{\ddot{R}}{R} + 8\pi G f \dot{C}^2 \frac{\ddot{R}}{R} + \Lambda = 0 \quad (11) \]

III. SOLUTION OF THE FIELD EQUATIONS:

Following Hoyle and Narlikar, the source equation of } C\text{-field: } C_{ij} = n_i f \text{ leads to } C = t, \text{ for large values of } t. \text{ Thus } C = 1. \text{ Using } \dot{C} = 1 \text{ in equations (5) and (10), we have

\[ \frac{(n-1)(n-2)}{2} \frac{\ddot{R}}{R^2} + \frac{(n-1)(n-2)}{2} \frac{k^2}{R^2} = 8\pi G \left( \rho - \frac{1}{2} f \right) + \Lambda, \quad (12) \]

\[ \frac{(n-1)(n-2)}{2} \frac{\ddot{R}}{R^2} + \frac{(n-1)(n-2)}{2} \frac{k^2}{R^2} = 4\pi G f + \Lambda, \quad (13) \]

To get deterministic solution in terms of cosmic time } t \text{ , we assume that } \Lambda = \frac{1}{R^2} \text{ [18].}

Solving equations (12) and (13), we get
\[
\frac{R}{R^2 + (n-2) \frac{R}{R^2} + \frac{(n-2)^2}{(n-1)(n-2)} R^2} = \frac{8 \pi G \rho}{(n-2)}.
\]

Using \( \Lambda = \frac{1}{R^2} \) in equation (13), we get

\[
(n-2) \frac{R}{R^2} + \frac{(n-2)^2}{2} R^2 + \frac{((n-2)(n-3)k-2)}{2} R^2 = 4 \pi Gf
\]

which again leads to

\[
(n-2) \frac{R}{R^2} + \frac{(n-2)(n-3)k}{2} R^2 + \frac{((n-2)(n-3)k-2)}{2} R^2 = 4 \pi GfR
\]

To find the solution of equation (16), let \( \frac{R}{R} = F(R) \)

which implies \( \frac{R}{F} = \frac{F}{R} \), where \( F = \frac{dF}{dR} \).

Using equation (17), equation (16) leads to

\[
\frac{dF}{R} + \frac{(n-3)}{2} F = \frac{8 \pi GfR}{(n-2)} - \frac{(n-2)(n-3)k-2}{2} R
\]

On integration equation (18) simplifies to

\[
F^2 = \frac{8 \pi GfR}{(n-2)(n-1)} R^2 - \frac{(n-2)(n-3)k-2}{2} R
\]

The integration constant taken to be zero for simplicity.

Using \( \frac{R}{R} = F(R) \), equation (19) simplifies to

\[
\frac{dR}{\sqrt{R^2 + \alpha^2}} = b dt,
\]

where

\[
\alpha = \frac{2(n-1)-(n-1)(n-2)(n-3)k}{(n-3)8 \pi Gf}, \quad b = \frac{8 \pi G}{(n-1)(n-2)}
\]

Equation (21) on integration gives

\[
R = \alpha \sin \beta t.
\]

In particular for \( \beta = 1 \), we have

\[
R = \sqrt{\frac{2}{(n-2)(n-3)k}} \sinh^2 t
\]

and

\[
\Lambda = \frac{1}{R} = \sqrt{\frac{(n-2)(n-3)}{2-n-2(n-3)k}} \cosh^2 t.
\]

Using equation (23), (24) in equation (12), we have

\[
\rho = \frac{1}{8 \pi G} \left[ \frac{(n-1)(n-2)}{2} \frac{R}{R^2} + \frac{(n-1)(n-2)k-2}{2} R^2 + 4 \pi Gf \right]
\]

Using equation (23) in metric (1), we get

\[
ds^2 = dt^2 - \left[ \frac{2}{(n-2)(n-3)} \right] \sinh^2 t \left[ \frac{dr^2 + r^2 (d\phi_{-2})^2}{1-k_r r^2} \right]
\]

Now we consider three different cases:

**Case (i): Flat Model \( k = 0 \)**

In this case, equation (20) leads to

\[
\frac{dR}{\sqrt{R^2 + \frac{2}{(n-2)(n-3)}}} = dt,
\]

Using equations (26) and (29) in equation (11), we have

\[
\frac{dC^2}{dt} + 2(n-1) \coth C^2 = 2(n-1) \coth t
\]

Equation (30) leads to

\[
C^2 \sinh^{2(n-1)} t = 2(n-1) \coth t \sinh^{2(n-1)} t dt.
\]

On simplification equation (31) reduce to

\[
C^2 = 1,
\]

which leads to

\[
C = t
\]

where \( C = t \),

which agrees with the value used in the source equation. Thus creation field \( C \) increases with time.

Using equation (27), Flat cosmological model \( k = 0 \), leads to

\[
ds^2 = dt^2 - \left[ \frac{2}{(n-2)(n-3)} \right] \sinh^2 t \left[ dr^2 + r^2 (d\phi_{-2})^2 \right].
\]

The Energy mass density \( \rho \), the cosmological constant \( \Lambda \) and the deceleration parameter \( q \) becomes

\[
\rho = \frac{(n-2)}{8 \pi G} \left[ 1 + \coth^2 t \right],
\]

\[
\Lambda = \frac{(n-2)(n-3)}{2} \coth^2 t,
\]

\[
R = \frac{2}{(n-2)(n-3) \sinh t},
\]

where \( 8 \pi G = (n-1)(n-2) \) and \( \beta = 1 \) assumed.

For Flat model (34), the energy density \( \rho \) is positive for all values of cosmic time \( t \) if \( n \neq 2 \). The deceleration parameter \( q < 0 \) indicating that the model is accelerating.
Case (ii): Closed Model \((k = 1)\)

In this case, equation (20) leads to
\[
\frac{dR}{\sqrt{R^2 + \frac{2 - (n - 2)(n - 3)}{(n - 2)(n - 3)}}} = dt , \quad (39)
\]
where \(\beta = \frac{8\pi Gf}{(n-1)(n-2)} = 1\).

Equation (39) on integration gives
\[
R = C_2 \sinh t , \quad (40)
\]
where \(C_2 = \frac{2 - (n - 2)(n - 3)}{2(n - 2)(n - 3)}\).

Using equations (26) and (40) in equation (11), we have
\[
\frac{dC^2}{dt} + 2(n - 1) \coth t C^2 = 2(n - 1) \coth t . \quad (41)
\]
Equation (41) leads to
\[
C^2 \sinh^{2(n-1)} t = 2(n-1) \int \coth t \, \sinh^{2(n-1)} t \, dt . \quad (42)
\]

On simplification equation (42) reduce to
\[
C = 1, \quad (43)
\]
which leads to
\[
C = t , \quad (44)
\]
which agrees with the value used in the source equation. Thus creation field \(C\) increases with time \(t\).

Using equation (27), Closed cosmological model \((k = 1)\), leads to
\[
ds^2 = dt^2 - \left(\frac{2 - (n - 2)(n - 3)}{(n - 2)(n - 3)}\right) \sinh^2 t \left(\frac{dr^2}{1 - r^2} + r^2 (d\phi_{n-2})^2\right) . \quad (45)
\]

The Energy mass density \(\rho\), the cosmological constant \((\Lambda)\) and the deceleration parameter \((q)\) becomes
\[
\rho = \frac{1}{8\pi Gf} \left[\frac{2(n - 2) \coth^2 t + 2(n - 2)}{2(n - 2)(n - 3)}\right] , \quad (46)
\]
\[
\Lambda = \frac{(n-2)(n-3)}{2(n-2)(n-3)} \coth^2 t , \quad (47)
\]
\[
R = \sqrt{\frac{2 - (n - 2)(n - 3)}{(n - 2)(n - 3)}} \sinh t , \quad (48)
\]
\[
q = -\tanh^2 t , \quad (49)
\]
where \(8\pi Gf = (n-1)(n-2)\) and \(\beta = 1\) assumed.

For Closed model (45), the energy density \(\rho\) is positive for all values of cosmic time \(t\) if \(n \neq 2; 1 < n < 4\). The deceleration parameter \(q < 0\) indicating that the model is accelerating.

Case (iii): Open Model \((k = -1)\)

In this case, equation (20) leads to
\[
\frac{dR}{\sqrt{R^2 + \frac{2 + (n - 2)(n - 3)}{(n - 2)(n - 3)}}} = dt , \quad (50)
\]
where \(\beta = \frac{8\pi Gf}{(n-1)(n-2)} = 1\).

Equation (50) on integration gives
\[
R = C_3 \sinh t , \quad (51)
\]
where \(C_3 = \frac{2 + (n - 2)(n - 3)}{2(n - 2)(n - 3)}\).

Using equations (26) and (51) in equation (11), we have
\[
\frac{dC^2}{dt} + 2(n - 1) \coth t C^2 = 2(n - 1) \coth t . \quad (52)
\]
Equation (52) leads to
\[
C^2 \sinh^{2(n-1)} t = 2(n-1) \int \coth t \, \sinh^{2(n-1)} t \, dt . \quad (53)
\]
On simplification equation (53) reduce to
\[
C = 1, \quad (54)
\]
which leads to
\[
C = t , \quad (55)
\]
which agrees with the value used in the source equation. Thus creation field \(C\) increases with time \(t\).

Using equation (27), Open cosmological model \((k = -1)\), leads to
\[
ds^2 = dt^2 - \left(\frac{2 + (n - 2)(n - 3)}{(n - 2)(n - 3)}\right) \sinh^2 t \left(\frac{dr^2}{1 + r^2} + r^2 (d\phi_{n-2})^2\right) . \quad (56)
\]

The Energy mass density \(\rho\), the cosmological constant \((\Lambda)\) and the deceleration parameter \((q)\) becomes
\[
\rho = \frac{1}{8\pi Gf} \left[\frac{2(n - 2) \coth^2 t + 2(n - 2)}{2(n - 2)(n - 3)}\right] , \quad (57)
\]
\[
\Lambda = \frac{(n-2)(n-3)}{2 + (n-2)(n-3)} \coth^2 t , \quad (58)
\]
\[
R = \sqrt{\frac{2 + (n - 2)(n - 3)}{(n - 2)(n - 3)}} \sinh t , \quad (59)
\]
\[
q = -\tanh^2 t , \quad (60)
\]
where \(8\pi Gf = (n-1)(n-2)\) and \(\beta = 1\) assumed.

For Open model (56), the energy density \(\rho\) is positive for all values of cosmic time \(t\) if \(n \neq 2; n^2 - 5n + 8 > 0\). The deceleration parameter \(q < 0\) indicating that the model is accelerating.
IV. CONCLUSION

N-dimensional FRW cosmological model has been obtained with time dependent $\Lambda$ in Creation field theory of gravitation when the universe is filled with dust distribution. Our model is the extension work of Bali & Saraf [25] for FRW space times. In this model, the spatial volume $V$ increases with the cosmic time $t$. For flat $(k = 0)$, closed $(k = 1)$ and open $(k = -1)$ models, the deceleration parameter $q < 0$ representing accelerating phase of Universe which matches with the recent SNeIa observations. Also, the energy density $\rho$ is positive. The creation field $C$ increases with time $t$ for all values of $k$ which matches with the result obtained by Hoyle-Narlikar's theory of gravitation.

REFERENCES


